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We present an analysis of compactifications of the type *IIB* superstring on $AdS_5 \times S^5/\Gamma$, where Γ is an abelian cyclic group. Every $\Gamma = Z_n$ of order $n \leq 12$ is considered. This results in 60 chiral models, and a systematic analysis with $n < 8$ yields four containing the minimal *SUSY* standard model with three families. One of these models extends to an infinite sequence of three-family *MSSMs*. We also give a lower bound on the number of chiral models for all values of n .

Orbifolded $AdS_5 \times S^5$ is fertile ground for building models which can potentially test string theory. When one bases the models on the conformal field theory gotten from the large N expansion of the AdS/CFT correspondence [1], stringy effects can show up at the scale of a few TeV . The first three-family model of this type had $\mathcal{N} = 1$ $SUSY$ and was based on a Z_3 orbifold [2]. However, since then the most studied examples have been models without supersymmetry based on both abelian [3], [4], [5] and non-abelian [6], [7] orbifolds of $AdS_5 \times S^5$. Here we return to Z_n orbifolds with supersymmetry, and systematically study those cases with chiral matter (*i.e.*, those with an imbalance between chiral supermultiplets and anti-chiral supermultiplets). We classify all cases up to $n \leq 12$, and show that several of these contain the minimal supersymmetric standard model ($MSSM$) with three families.

Let us summarize how these models are constructed (for a review see [7]). First we choose a discrete group Γ with which to form the orbifold $AdS_5 \times S^5/\Gamma$. The replacement of S^5 by S^5/Γ reduces the supersymmetry to $\mathcal{N} = 0, 1$ or 2 from the initial $\mathcal{N} = 4$, depending on how Γ is embedded in the $SO(6) \sim SU(4)$ isometry of S^5 . The case of interest here is $\mathcal{N} = 1$ $SUSY$ where Γ completely embeds in an $SU(3)$ subgroup of the $SU(4)$ isometry. *I.e.*, we embed $\text{rep}(\Gamma) \rightarrow \mathbf{4}$ of $SU(4)$ as $\mathbf{4} = (\mathbf{1}, \mathbf{r})$ where $\mathbf{1}$ is the trivial irrep of Γ and \mathbf{r} is a nontrivial (but possibly reducible) three dimensional representation of Γ . The chiral supermultiples generated by this embedding are given by

$$\sum_i \mathbf{4} \otimes R_i \quad (1)$$

where the set $\{R_i\}$ runs over all the irreps of Γ . For our choice, $\Gamma = Z_n$, the irreps are all one dimensional and as a consequence of the choice of N in the $1/N$ expansion, the gauge group [8] is $SU^n(N)$. Chiral models require the $\mathbf{4}$ to be complex ($\mathbf{4} \neq \mathbf{4}^*$) while a proper embedding requires $\mathbf{6} = \mathbf{6}^*$ where $\mathbf{6} = (\mathbf{4} \otimes \mathbf{4})_{\text{antisym}}$, (even though the $\mathbf{6}$ does not enter the model). This information is sufficient for us to begin our investigation. We will choose $N = 3$ throughout, and if we use the fact that $SU_L(2)$ and $U_Y(1)$ are embedded in diagonal subgroups $SU^p(3)$ and $SU^q(3)$ of the initial $SU^n(3)$, the ratio $\frac{\alpha_2}{\alpha_Y}$ turns out to be $\frac{p}{q}$. This implies the initial value of θ_W is calculable in these models and $\sin^2 \theta_W$ satisfies

$$\sin^2 \theta_W = \frac{3}{3 + 5 \left(\frac{p}{q} \right)}. \quad (2)$$

On the other hand, a more standard approach is to break the initial $SU^n(3)$ to $SU_C(3) \otimes SU_L(3) \otimes SU_R(3)$ where $SU_L(3)$ and $SU_R(3)$ are embedded in diagonal subgroups $SU^p(3)$ and $SU^q(3)$ of the initial $SU^n(3)$. We then embed all of $SU_L(2)$ in $SU_L(3)$ but $\frac{1}{3}$ of $U_Y(1)$ in $SU_L(3)$ and the other $\frac{2}{3}$ in $SU_R(3)$. This modifies the $\sin^2 \theta_W$ formula to:

$$\sin^2 \theta_W = \frac{3}{3 + 5 \left(\frac{\alpha_2}{\alpha_Y} \right)} = \frac{3}{3 + 5 \left(\frac{3p}{p+2q} \right)} \quad (3)$$

Note, this coincides with the previous result when $p = q$. We will use the later result when calculating $\sin^2 \theta_W$ below. A similar relation holds for Pati–Salam type models [9].

First a Z_2 orbifold has only real representations and therefore will not yield a chiral model. (Note, although all matter is in chiral supermultiplet, if there is a left-handed supermultiplet to match each right-handed supermultiplet, then the model has no overall chirality, *i.e.*, it is vectorlike.)

Next, for $\Gamma = Z_3$ the choice $\mathbf{4} = (\mathbf{1}, \alpha, \alpha, \alpha)$ with $N = 3$ where $\alpha = e^{\frac{2\pi i}{3}}$ (in what follows we will write $\alpha = e^{\frac{2\pi i}{n}}$ for Z_n), yields the three family trinification [10] model of [2], but without sufficient scalars to break the gauge symmetry

to the MSSM. Here the initial value of $\sin^2 \theta_W = \frac{3}{8}$, so unification at the TeV scale is also problematic. There is another chiral model for $\mathbf{4} = (\mathbf{1}, \alpha, \alpha, \alpha^2)$ but it can have at most one chiral family.

Z_4 orbifolds allow only one chiral model with $\mathcal{N} = 1$ *SUSY*. It is generated by $\mathbf{4} = (\mathbf{1}, \alpha, \alpha, \alpha^2)$ but can have at most two chiral families.

There are two chiral models for Z_5 , and they are fixed by choosing $\mathbf{4} = (\mathbf{1}, \alpha, \alpha, \alpha^3)$ and $\mathbf{4} = (\mathbf{1}, \alpha, \alpha^2, \alpha^2)$. Before looking at these in detail, let us pause to define a useful notation for classifying models. The initial model (before any symmetry breaking) is completely fixed (recall we always are taking $N = 3$) by the choice of Z_n and the embedding $\mathbf{4} = (\mathbf{1}, \alpha^i, \alpha^j, \alpha^k)$, so we define the model to be M_{ijk}^n . We immediately observe that the conjugate model $M_{n-i, n-j, n-k}^n$ contains the same information, so we need not study it separately.

Returning now to Z_5 , the two models are M_{113}^5 and M_{122}^5 . (Other inconsistent models are eliminated by requiring $\mathbf{6} = \mathbf{6}^*$ keeping the number of models limited.) We find no pattern of spontaneous symmetry breaking (*SSB*) for M_{113}^5 that yields the *MSSM*, but M_{122}^5 is more interesting. The matter content of M_{122}^5 is shown in Table 1.

M_{122}^5	1	α	α^2	α^3	α^4
1	\times	\times	$\times \times$		
α		\times	\times	$\times \times$	
α^2			\times	\times	$\times \times$
α^3	$\times \times$			\times	\times
α^4	\times	$\times \times$			\times

Table 1: Matter content for the model M_{122}^5 . The $\times \times$ entry at the $(1, \alpha^2)$ position means the model contains $2(3, 1, \bar{3}, 1, 1)$ of $SU^5(3)$, etc. The diagonal entries are $(8 + 1, 1, 1, 1, 1)$, etc.

A vacuum expectation value (*VEV*) for $(3, \bar{3}, 1, 1, 1)$ breaks the symmetry to $SU_D(3) \otimes SU_3(3) \otimes SU_4(3) \otimes SU_5(3)$ and a further *VEV* for $(1, 3, \bar{3}, 1)$ breaks the symmetry to $SU_D(3) \otimes SU_{D'}(3) \otimes SU_5(3)$. Identifying $SU_C(3)$ with $SU_D(3)$, embedding $SU_L(2)$ in $SU_{D'}(3)$ and $U_Y(1)$ partially in $SU_5(3)$ and partially in $SU_{D'}(3)$ gives an initial value of $\sin^2 \theta_W = \frac{2}{7} = .286$, and implies a unification scale around $2 \times 10^7 GeV$.

The remaining chiral multiplets are

$$3[(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3)] \quad (4)$$

We have sufficient octets to continue the symmetry breaking all the way to $SU(3) \otimes SU(2) \otimes U(1)$, and so arrive at the *MSSM* with three families (plus additional vector-like matter that is heavy and therefore not in the low energy spectrum).

Before analyzing more models in detail, it is useful to tabulate the possible model for each value of n . To this end, note we always have a proper embedding (*i.e.*, $\mathbf{6} = \mathbf{6}^*$) for $\mathbf{4} = (\mathbf{1}, \alpha^i, \alpha^j, \alpha^k)$ when $i + j + k = n$. To show this we use the fact that the conjugate model has $i \rightarrow i' = n - i$, $j \rightarrow j' = n - j$ and $k \rightarrow k' = n - k$. Summing we find $i' + j' + k' = 3n - (i + j + k) = 2n$. From $\mathbf{6} = (\mathbf{4} \otimes \mathbf{4})_{antisym}$ we find $\mathbf{6} = (\alpha^i, \alpha^j, \alpha^k, \alpha^{j+k}, \alpha^{i+k}, \alpha^{i+j})$, but $i + j = n - k = k'$. Likewise $i + k = j'$ and $j + k = i'$ so $\mathbf{6} = (\alpha^i, \alpha^j, \alpha^k, \alpha^{i'}, \alpha^{j'}, \alpha^{k'})$ and this is $\mathbf{6}^*$ up to an automorphism which is sufficient to provide vectorlike matter in this sector in the non-SUSY models and here provide a proper embedding. Models with $i + j + k = n$ (we will call these partition models) are always chiral, with total

chirality $\chi = 3N^2n$ except in the case where n is even and one of i, j , or k is $n/2$ where $\chi = 2N^2n$. (No more than one of i, j , and k can be $n/2$ since they add to n and are all positive.) This immediately gives us a lower bound on the number of chiral models at fixed n . It is the the number of partitions of n into three non-negative integers. There is another class of models with $i' = k$ and $j' = j^2$, and total chirality $\chi = N^2n$; for example a Z_9 orbifold with $\mathbf{4} = (\mathbf{1}, \alpha^3, \alpha^3, \alpha^6)$. And there are a few other sporadically occurring cases like M_{124}^6 , which typically have reduced total chirality, $\chi < 3N^2n$.

We now tabulate all the Z_n orbifold models up to $n = 12$ along with the total chirality of each model, (see Table 2).

n	$\mathbf{4}$	χ/N^2	comment
3	$(\mathbf{1}, \alpha, \alpha, \alpha)$	9	$i + j + k = 3$; one model ($i = j = k = 1$)
3	$(\mathbf{1}, \alpha, \alpha, \alpha^2)^*$	3	
4	$(\mathbf{1}, \alpha, \alpha, \alpha^2)$	8	$i + j + k = 4$; one model
5	$(\mathbf{1}, \alpha^i, \alpha^j, \alpha^k)$	15	$i + j + k = 5$; 2 models
6	$(\mathbf{1}, \alpha^i, \alpha^j, \alpha^k)$	12	$i + j + k = 6$; 3 models
6	$(\mathbf{1}, \alpha, \alpha^2, \alpha^4)^*$	6	
6	$(\mathbf{1}, \alpha^2, \alpha^2, \alpha^4)^*$	6	
7	$(\mathbf{1}, \alpha^i, \alpha^j, \alpha^k)$	21	$i + j + k = 7$; 4 models
8	$(\mathbf{1}, \alpha^i, \alpha^j, \alpha^k)$	≤ 24	$i + j + k = 8$; 5 models
9	$(\mathbf{1}, \alpha^i, \alpha^j, \alpha^k)$	27	$i + j + k = 9$; 7 models
9	$(\mathbf{1}, \alpha, \alpha^4, \alpha^7)^*$	27	
9	$(\mathbf{1}, \alpha^3, \alpha^3, \alpha^6)^*$	9	
10	$(\mathbf{1}, \alpha^i, \alpha^j, \alpha^k)$	30	$i + j + k = 10$; 8 models
11	$(\mathbf{1}, \alpha^i, \alpha^j, \alpha^k)$	33	$i + j + k = 11$; 10 models
12	$(\mathbf{1}, \alpha^i, \alpha^j, \alpha^k)$	≤ 36	$i + j + k = 12$; 12 models
12	$(\mathbf{1}, \alpha^2, \alpha^4, \alpha^8)^*$	12	
12	$(\mathbf{1}, \alpha^4, \alpha^4, \alpha^8)^*$	12	

Table 2. All chiral Z_n orbifold models with $n \leq 12$. Three of the $n = 8$ models have $\chi/N^2 = 24$; the other two have $\chi/N^2 = 16$. Of the 12 models with $i + j + k = 12$, three have models $\chi/N^2 = 24$ and the other nine have $\chi/N^2 = 36$. Of the 60 models 53 are partition models, while the remaining 7 models that do not satisfy $i + j + k = n$, are marked with an asterisk (*).

We have analyzed all the models for Z_6 orbifolds, and find only one of phenomenological interest. It is M_{123}^6 , where the matter multiplets are shown in Table 3.

M_{123}^6	1	α	α^2	α^3	α^4	α^5
1	×	×	×	×		
α		×	×	×	×	
α^2			×	×	×	×
α^3	×			×	×	×
α^4	×	×			×	×
α^5	×	×	×			×

Table 3: Chiral supermultiplets for the model M_{123}^6 .

VEV s for $(3, \bar{3}, 1, 1, 1, 1)$, $(1, 1, 3, \bar{3}, 1, 1)$ and $(1, 1, 1, 1, 3, \bar{3})$ break the symmetry to $SU_{12}(3) \otimes SU_{34}(3) \otimes SU_{56}(3)$ where $SU_{12}(3)$ is the diagonal subgroup of $SU_1(3) \otimes SU_2(3)$, etc., and the remaining chirality resides in $3[(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3)]$. Again, we have octets of all six initial $SU(3)$ s, so we can break to a three-family $MSSM$, but with $\sin^2 \theta_W = \frac{3}{8}$. There is no other pattern of SSB that gives three families.

For Z_7 , we again find only one model that can break to a three-family $MSSM$. It is M_{133}^7 , with matter shown in Table 4.

M_{133}^7	1	α	α^2	α^3	α^4	α^5	α^6
1	×	×		×	×		
α		×	×		×	×	
α^2			×	×		×	×
α^3				×	×		×
α^4	×	×			×	×	
α^5		×	×			×	×
α^6	×		×	×			×

Table 4: Chiral supermultiplets for the model M_{133}^7 .

First VEV s for $(3, \bar{3}, 1, 1, 1, 1, 1)$ and $(1, 1, 3, \bar{3}, 1, 1, 1)$ breaks the symmetry to $SU_{12}(3) \otimes SU_{34}(3) \otimes SU_5(3) \otimes SU_6(3) \otimes SU_7(3)$. Then a VEV for $(1, 1, 1, 3, \bar{3})$ breaks this to $SU_{12}(3) \otimes SU_{34}(3) \otimes SU_5(3) \otimes SU_{67}(3)$, and leaves the following multiplets chiral

$$(3, \bar{3}, 1, 1) + (1, 3, \bar{3}, 1) + (1, 1, 3, \bar{3}) + (\bar{3}, 1, 1, 3) + 2[(3, \bar{3}, 1, 1) + (1, 3, 1, \bar{3}) + (\bar{3}, 1, 1, 3)] \quad (5)$$

Finally, a VEV for $(1, 3, \bar{3}, 1)$ yields the $MSSM$ with three chiral families. Identifying $SU_C(3)$ with $SU_{12}(3)$ and embedding $SU_L(2)$ in $SU_{67}(3)$ and $U_Y(1)$ in $SU_{345}(3)$ gives $\sin^2 \theta_W = \frac{7}{22} = .318$ and implies a unification scale around $10^{10} GeV$.

The $n > 7$ models can be analyzed in a similar manner. The total number of models grows with n . There are also potentially interesting examples for $N > 4$. Although we have not made a systematic study of the models with $n \geq 8$,

we close with a rather compact example of a three-family $MSSM$ at $n = 9$. The model is M_{123}^6 with matter given in Table 5.

M_{333}^9	1	α	α^2	α^3	α^4	α^5	α^6	α^7	α^8
1	\times			$\times \times \times$					
α		\times			$\times \times \times$				
α^2			\times			$\times \times \times$			
α^3				\times			$\times \times \times$		
α^4					\times			$\times \times \times$	
α^5						\times			$\times \times \times$
α^6	$\times \times \times$						\times		
α^7		$\times \times \times$						\times	
α^8			$\times \times \times$						\times

Table 5: Chiral supermultiplets for the model M_{333}^9 .

VEV s for the octets of $SU_k(3)$, where $k = 2, 3, 5, 6, 8$, and 9 breaks the symmetry to $SU_1(3) \otimes SU_4(3) \otimes SU_7(3)$. (Each chiral supermultiplet of representation R contains one chiral fermion multiplet in representation R , and two scalar (we need not distinguish scalars from pseudoscalars here) multiplets in representation R . Therefore, there are two scalar octets for each $SU_k(3)$. When one octet of $SU_k(3)$ is given a VEV , gauge freedom can be used to diagonalize that VEV . However, there is not enough gauge freedom left to diagonalize the VEV of the second octet of the same $SU_k(3)$. Therefore $SU_k(3)$ can be broken completely by $SU_k(3)$ s for the two octets). The chirality remaining after this octet breaking is $3[(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3)]$. Further symmetry breaking via single octets of $SU_3(3)$ and $SU_7(3)$ leads us to the three-family $MSSM$, but with $\sin^2 \theta_W = \frac{3}{8}$. Note that any model of the type $M_{\frac{n}{3} \frac{n}{3} \frac{n}{3}}^n$ can be handled this way, and can lead to a three-family $MSSM$. Hence this provides an infinite class of three-family models.

We have found 60 chiral Z_n orbifolds for $n \leq 12$. A systematic search up through $n = 7$ yields four models that can result in three-family minimal supersymmetric standard models. They are M_{111}^3 , M_{122}^5 , M_{123}^6 , and M_{133}^7 . We suspect there are many more models with sensible phenomenology at larger n , and we have pointed out one example M_{333}^9 , which is particularly simple in its spontaneous symmetry breaking, and is also a member of an infinite series of models $M_{\frac{n}{3} \frac{n}{3} \frac{n}{3}}^n$, which all can lead to three-family $MSSM$ s. Orbifolded AdS/CFT models hold great promise for testing string theory not far above the TeV scale, and they have inspired models [11] with phenomenology ranging from light magnetic monopoles [12] to an anomalous muon magnetic moment [13]. They have also provided a check on higher loop β functions [14], and raised interesting cosmological questions [15].

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